



EE 232 Lightwave Devices Lecture 10: Intersubband Absorption in Quantum Wells

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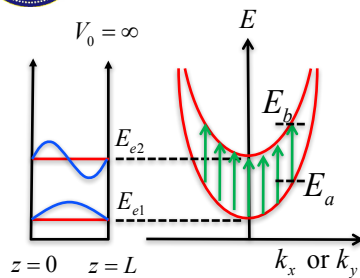
University of California, Berkeley
Electrical Engineering and Computer Sciences Dept.

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Intersubband Transition in Quantum Wells



For a 10-nm QW
in GaAs

$$\begin{aligned} E_{e1}^{e2} &= E_{e2} - E_{e1} \\ &= 224 \text{ meV} - 56 \text{ meV} \\ &= 168 \text{ meV} \end{aligned}$$

$$\lambda = \frac{1.24}{0.168} = 7.4 \mu\text{m}$$

- Transition between quantized energy levels in a quantum well by absorption or emission of a photon

- Absorption

- Infrared photodetectors
- Thermal imager: blackbody radiation of human body $\sim 10 \mu\text{m}$
- 3 ~ 5 and 8 ~ 10 μm wavelength bands particularly interesting

- Emission

- Gain media for quantum cascaded lasers (QCL)
- Long wavelength emission coincides with molecular vibration spectra

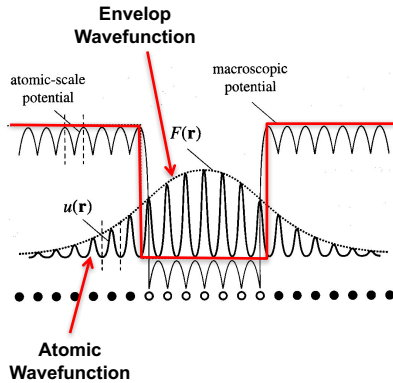
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Optical Matrix Element for Intersubband Transition

Quantum Well Wavefunction



$$|a\rangle = \psi_a(\vec{r}) = u_c(\vec{r}) \frac{e^{i\vec{k}_t \cdot \vec{\rho}}}{\sqrt{A}} \phi_1(z)$$

$$|b\rangle = \psi_b(\vec{r}) = u_c(\vec{r}) \frac{e^{i\vec{k}_t \cdot \vec{\rho}}}{\sqrt{A}} \phi_2(z)$$

$$H'_{ba} = -\vec{E} \cdot \vec{\mu}_{ba}$$

$$\vec{\mu}_{ba} = \langle b | e\vec{r} | a \rangle$$

$$\approx \langle u_c(\vec{r}) | u_c(\vec{r}) \rangle \int \frac{e^{i\vec{k}_t \cdot \vec{\rho}}}{\sqrt{A}} \frac{e^{i\vec{k}_t \cdot \vec{\rho}}}{\sqrt{A}} d\vec{\rho} \cdot \int \phi_2^*(z) e\vec{r} \phi_1(z) dz$$

Slowly Varying Envelop Approx

$$= 1 \cdot \delta_{\vec{k}_t, \vec{k}_t} \hat{\mu}_{21} z$$

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Absorption Coefficient for Intersubband Transition

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} \cdot \frac{2}{V} \sum_{\vec{k}_t} g(E_b - E_a - \hbar\omega) \left| \hat{e} \cdot \vec{\mu}_{ba} \right|^2 (f_a - f_b)$$

The summation is over all electron states: $\vec{k}_t = k_x \hat{x} + k_y \hat{y}$

We need to consider the finite width of the energy spread

(otherwise the absorption is a delta function with infinite absorption peak)

$$g(\Delta E) = \frac{1}{\pi} \frac{\Gamma/2}{\Delta E^2 + (\Gamma/2)^2} \quad (\text{Lorentzian Lineshape})$$

$$\left| \hat{e} \cdot \vec{\mu}_{ba} \right|^2 = \left| \mu_{21} \right|^2 \quad \text{only when } \hat{e} = \hat{z} \quad (\text{TM polarization})$$

$$E_b - E_a = E_{e1}^2, \quad \text{independent of } \vec{k}_t$$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} g(E_{e1}^2 - \hbar\omega) \left| \mu_{21} \right|^2 \frac{2}{V} \sum_{\vec{k}_t} (f_a - f_b)$$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} g(E_{e1}^2 - \hbar\omega) \left| \mu_{21} \right|^2 (N_1 - N_2)$$

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Absorption Coefficient for Intersubband Transition

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} g(E_{e1}^{e2} - \hbar\omega) |\mu_{21}|^2 (N_1 - N_2)$$

(1) $E_1 < F < E_2$

$$N_2 \approx 0$$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} g(E_{e1}^{e2} - \hbar\omega) |\mu_{21}|^2 N_1$$

is proportional to doping concentration

(2) $E_2 < F$

$$N_1 = \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \ln(1 + e^{\frac{F - E_1}{k_B T}}) \approx \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \frac{F - E_1}{k_B T} = \frac{m_e^*}{\pi \hbar^2 L_z} (F - E_1)$$

$$N_2 \approx \frac{m_e^*}{\pi \hbar^2 L_z} (F - E_2)$$

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} g(E_{e1}^{e2} - \hbar\omega) |\mu_{21}|^2 \frac{m_e^*}{\pi \hbar^2 L_z} E_{e1}^{e2}$$

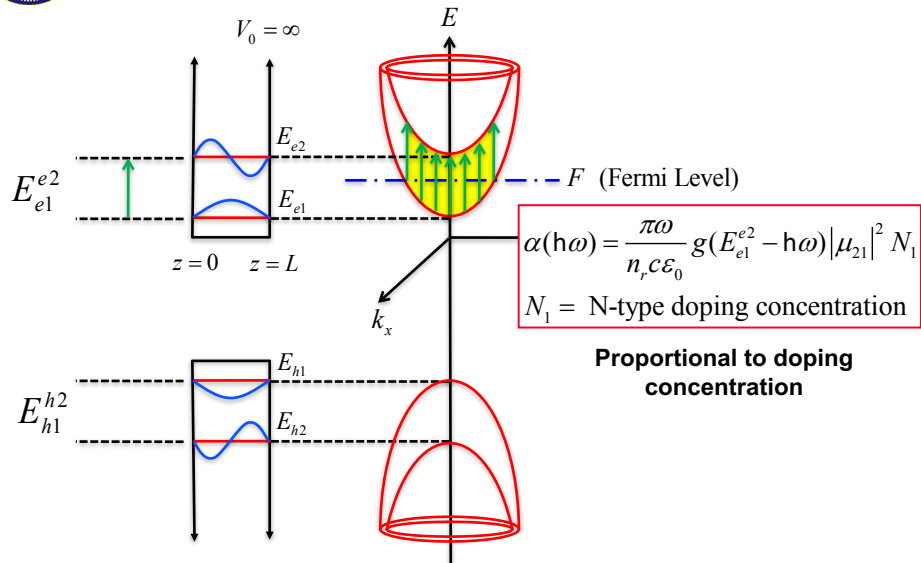
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is a constant independent of doping concentration

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Intersubband Transition in P-Doped QW (1): Fermi-Level Below Second Subband

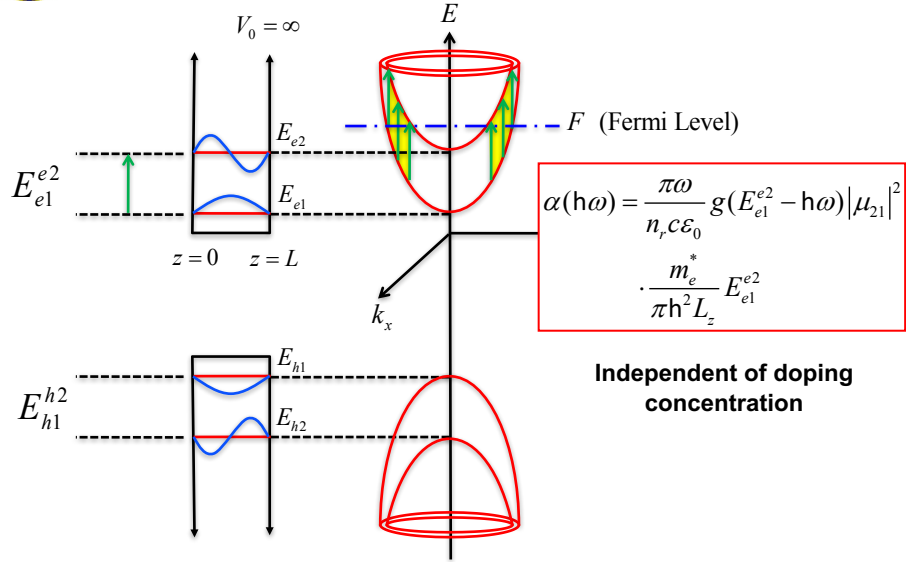


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Intersubband Transition in N-Doped QW (2): Fermi-Level Above Second Subband

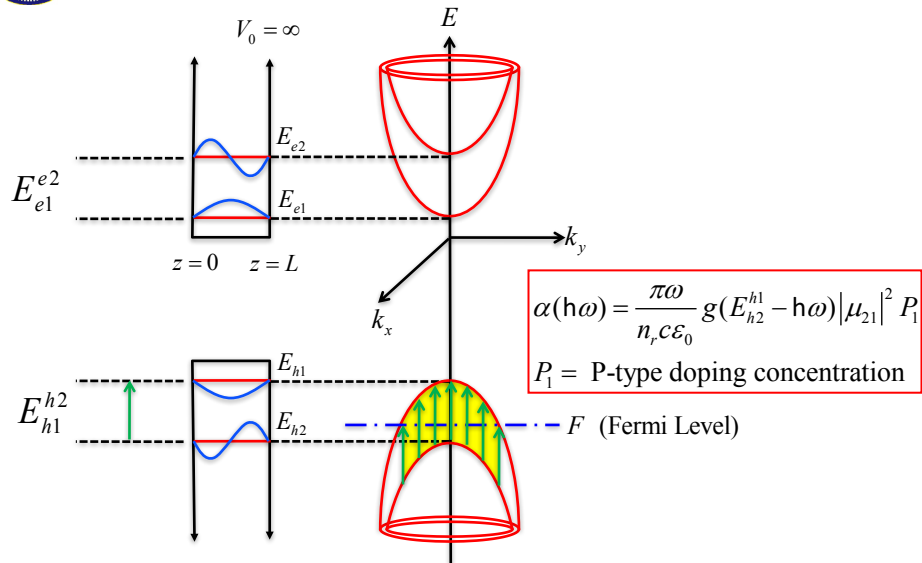


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Intersubband Transition in P-Doped QW (1): Fermi-Level Above Second Subband

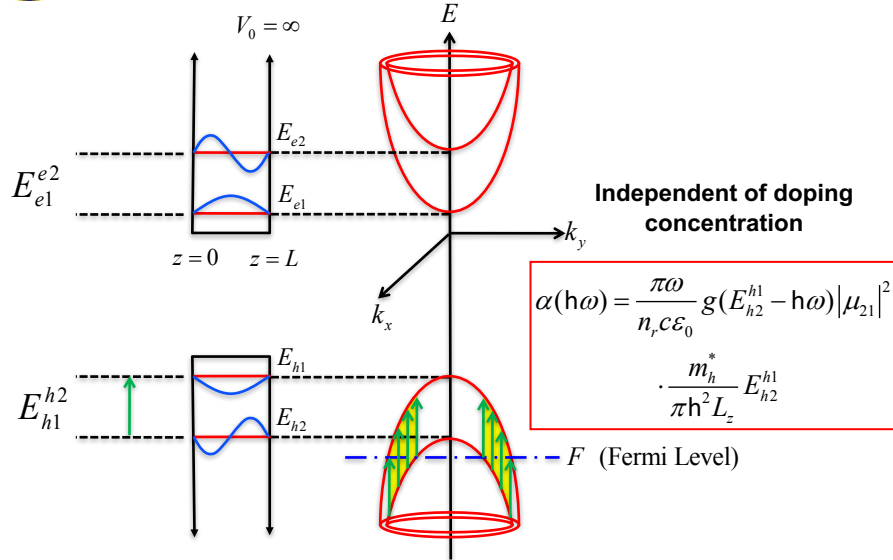


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Intersubband Transition in P-Doped QW (2): Fermi-Level Below Second Subband



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Intersubband Dipole Moment

$$\phi_1(z) = \sqrt{\frac{2}{L_z}} \sin\left(\frac{\pi}{L_z} z\right)$$

$$\phi_2(z) = \sqrt{\frac{2}{L_z}} \sin\left(\frac{2\pi}{L_z} z\right)$$

$$\mu_{21} = e \int_0^{L_z} \phi_2(z) \cdot z \cdot \phi_1(z) dz$$

$$= \frac{2e}{L_z} \int_0^{L_z} \sin\left(\frac{\pi}{L_z} z\right) \cdot z \cdot \sin\left(\frac{2\pi}{L_z} z\right) dz$$

$$= -\frac{16}{9\pi^2} eL_z$$

Compare with dipole moment of interband transition:

$$\frac{|\mu_{21}^{intra}|}{|\mu_{cv}^{inter}|} = \frac{16}{9\pi^2} \frac{eL_z}{er_{cv}} \approx \frac{0.2L_z}{0.4 \text{ nm}}$$

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Example

$$L_z = 10nm$$

$$E_{e1} = \frac{\hbar^2}{2m_e^*} \left(\frac{\pi}{L_z} \right)^2 = 56 \text{ meV}$$

$$E_{e2} = \frac{\hbar^2}{2m_e^*} \left(\frac{2\pi}{L_z} \right)^2 = 224 \text{ meV}$$

$$E_{e1}^{e2} = 168 \text{ meV}$$

$$N = 10^{18} \text{ cm}^{-3}$$

First, determine if the second subband is occupied.

Find $N_{1,\max}$, the electron concentration in the first subband

when the Fermi level is at the bottom of the second subband:

$$N_{1,\max} = \frac{m_e^*}{\pi \hbar^2 L_z} E_{e1}^{e2} = 4.7 \times 10^{18} \text{ cm}^{-3} > N = 10^{18} \text{ cm}^{-3}$$

So the second subband is not occupied. $N_2 = 0$

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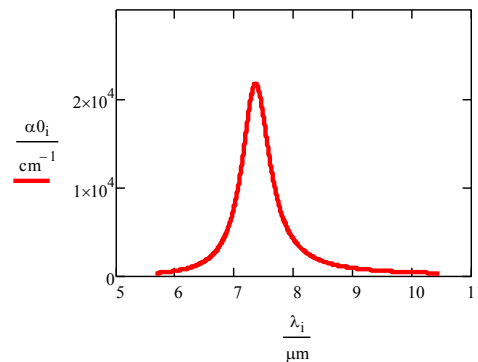


Absorption Spectra

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \epsilon_0} g(E_{e1}^{e2} - \hbar\omega) \left(\frac{16}{9\pi^2} eL_z \right)^2 N$$

Peak absorption

$$\alpha_{\max} = \frac{\pi\omega}{n_r c \epsilon_0} \frac{1}{\pi \left(\frac{\Gamma}{2} \right)} \left(\frac{16}{9\pi^2} eL_z \right)^2 N \approx 2 \times 10^4 \text{ cm}^{-1}$$

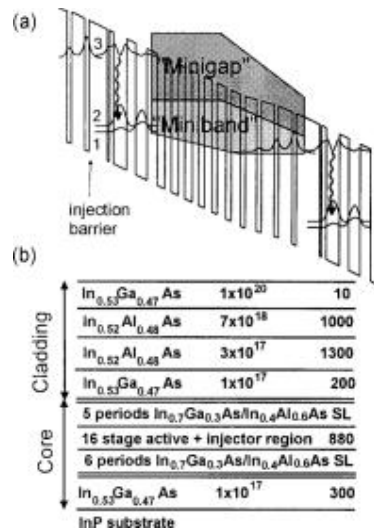


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Quantum Cascade Laser (QCL)



- Using intersubband transition to provide gain
- From THz to IR
- Key design
 - Upper state should be aligned with a “minigap” to prevent direct tunneling loss of upper state electrons
 - Lower state should be aligned with a “mini-band” to quickly remove the lower state population
- The electrons can be “reused” by cascading the quantum wells
- Emission wavelength tailorable by varying thickness of various layers